Exercise 76

When temperature is 0 degrees Celsius, the Fahrenheit temperature is 32. When the Celsius temperature is 100, the corresponding Fahrenheit temperature is 212. Express the Fahrenheit temperature as a linear function of C, the Celsius temperature, $F(C)$.

    a. Find the rate of change of Fahrenheit temperature for each unit change temperature of Celsius.

    b. Find and interpret $F(28)$.

    c. Find and interpret $F(-40)$.

Solution

The relationship between temperature scales is a linear one.

$$F(C) = mC + b$$

From the given information, two points on this line are $(0, 32)$ and $(100, 212)$. The first point says that when the input is $C = 0$, the output is $F = 32$.

$$32 = m(0) + b$$

The second point says that when the input is $C = 100$, the output is $F = 212$.

$$212 = m(100) + b$$

This is a system of two equations for two unknowns that can be solved for.

$$\begin{cases} b = 32 \\ 100m + b = 212 \end{cases}$$

Plug the value of b into the second equation.

$$100m + (32) = 212 \quad \rightarrow \quad 100m = 180 \quad \rightarrow \quad m = 1.8$$

Now that m and b have been solved for, the relationship between the Fahrenheit and Celsius temperature scales is known.

$$F(C) = 1.8C + 32$$

The slope, 1.8, is the rate of change of Fahrenheit temperature for each unit change temperature of Celsius. The Fahrenheit temperature corresponding to 28°C is

$$F(28) = 1.8(28) + 32 = 82.4,$$

and the Fahrenheit temperature corresponding to -40°C is

$$F(-40) = 1.8(-40) + 32 = -40.$$